Failed Power Domination on Knödel Graphs

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Outline

1. Introduction
   - Notation
   - Domination
   - Zero Forcing
   - Power Domination
   - Failed Power Domination
   - Applications

2. Results
   - General Extreme Values
   - Knödel Graphs
   - Complexity

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Notation

- Graphs
  - Made up of vertices and edges. Vertices are connected to each other with edges.
  - $G = (V, E)$, where V is the set of vertices, and E is the set of edges.
  - Directed vs. Undirected
    - We will only be working with undirected graphs
- $N(v) :=$ set of neighbors of a vertex

Domination

- DOMINATE(G, S)

```
Assuming S ⊆ V
for v ∈ S do
    for u ∈ N(v) do
        if u ∉ S then
            S = S ∪ {u}
        end if
    end for
end for
```

- After domination has been performed, if $S = V$, then the initial set S is a *dominating set* of G
- The *domination number* of a graph, or the cardinality of the smallest dominating set of a graph, is noted $\gamma(G)$

Domination Examples

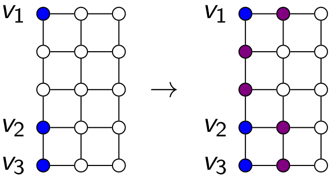

Figure 1: S is a not a dominating set.

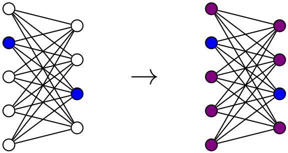

Figure 2: S is a dominating set

Zero Forcing

- ZERO_FORCE(G, S)

```
Assuming S ⊆ V
while S has not been changed do
    for v ∈ S do
        if Exactly one empty vertex in N(v) then
            Fill in that vertex and add it to S
        end if
    end for
end while
```

- If, after the zero forcing has been performed, $S = V$, then the initial set S is a *zero forcing set* of G.
- The *zero forcing number* of a graph is the cardinality of the smallest zero forcing set of a graph, and is noted F(G)

Zero Forcing Examples

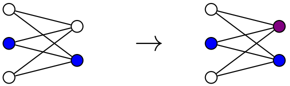

Figure 3: S is a not a zero forcing set.

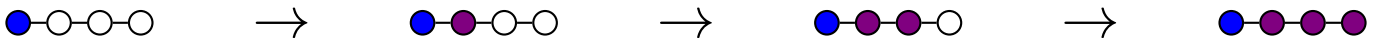

Figure 4: S is a zero forcing set.

Power Domination

- POWER_DOMINATE(G, S) {S will be updated at each step}
  1: DOMINATE(G, S)
  2: ZERO_FORCE(G, S)

- If, after the power domination has been performed, $S = V$, then the initial set S is a *power dominating set* of G.
- The *power domination number* of a graph is the **minimum** cardinality of such a power dominating set and is noted $\gamma_p(G)$

Power Domination Example

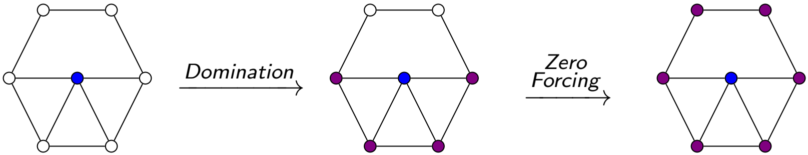

Figure 5: S is a power dominating set.

Failed Power Domination

- $\bar{\gamma}_p$
- The *failed power domination number* of a graph is the cardinality of the **largest** set that does not power dominate the graph.

Failed Power Domination Example

$$\bar{\gamma}_p(K_{5,3}) = 3$$

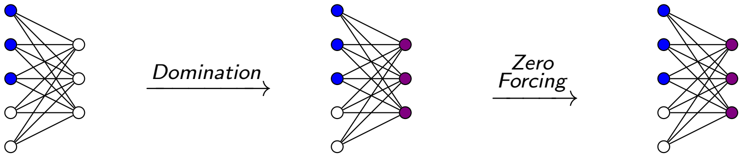

Figure 6: S is a failed power dominating set. If you add any more vertices to S, it will become a power dominating set.

PMUs

- Phasor Measurement Units
  - Monitor electric power networks
  - Expensive

- $\bar{\gamma}_p(G) + 1$ is the number of PMUs that you can put **anywhere** on the graph and monitor the entire graph.

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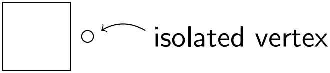
General Extreme Values

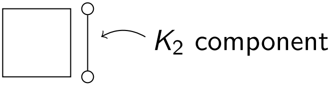
Let n=$|G.V|$

Let $\square$ denote a graph.

$\bar{\gamma}_p(G) = n - 1$ if and only if G has an isolated vertex.

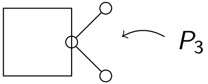


$\bar{\gamma}_p(G) = n - 2$ if and only if G contains K_2 as a component, and no isolated vertices.

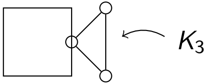


General Extreme Values

- $\bar{\gamma}_p(G) = n - 3$ if and only if G contains no components that are isolated vertices or K_2 and G contains a copy of
  - P_3 where only the middle vertex may be adjacent to other vertices in the $V(G)$

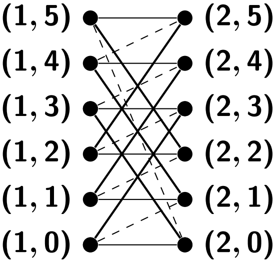


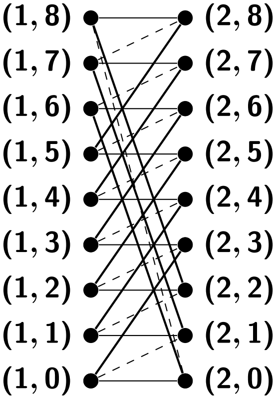
  - K_3 where at most one of the vertices may be adjacent to other vertices in $V(G)$



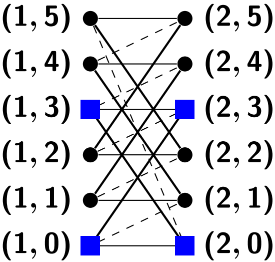
Knödel Graphs $W_{\#vertices,\ degree}$

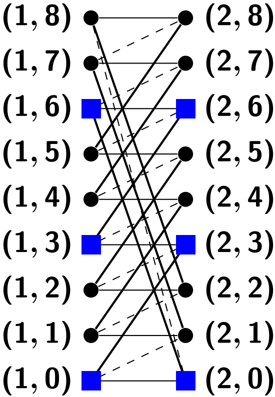
In this case $W_{6k,3}$ where k $\geq$ 2.



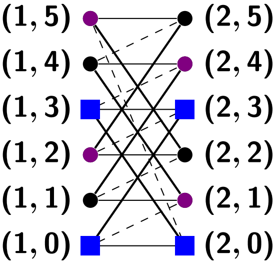


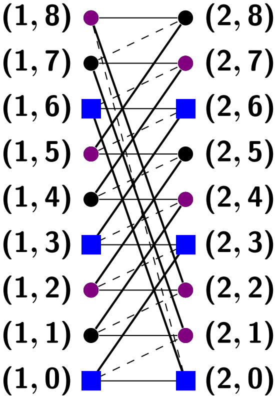
Knödel Graphs Initial Sets





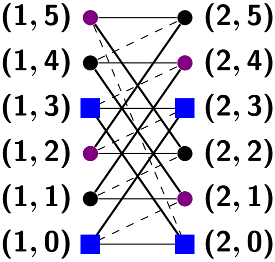
Knödel Graphs – Domination Step

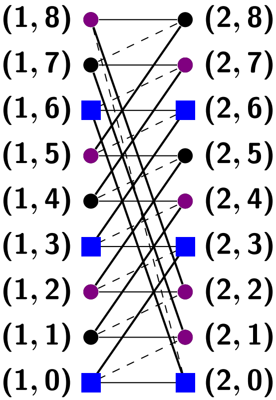




Knödel Graphs – Zero Forcing Step

We cannot fill in any more vertices. The Zero Forcing step is stalled and the initial set fails to power dominate.





Complexity

FAILED POWER DOMINATING SET (FPDS)
Instance: a graph $G = (V, E)$ and a positive integer k
Question: Does G have a proper stalled subset of cardinality at least k?

Theorem

FPDS is NP-complete.

Email me if you want to see the proof :-)

Thank you!

For more information, please do not hesitate to contact

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