Failed Power Domination: Computational Results, Extreme Values, and Complexity

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Outline

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Notation

- Graphs
  - Made up of vertices and edges. Vertices are connected to each other with edges.
  - $G = (V, E)$, where V is the set of vertices, and E is the set of edges.
  - Directed vs. Undirected
    - We will only be working with undirected graphs
- $N(v) :=$ set of neighbors of a vertex

Domination

- DOMINATE(G, S)

  Assuming $S \subseteq V$
  for $v \in S$ **do**
      for $u \in N(v)$ **do**
          if $u \notin S$ **then**
              $S = S \cup \{u\}$
          end if
      end for
  end for

- If, after the domination has been performed, $S = V$, then the initial set S is a *dominating set* of G
- The *domination number* of a graph, or the cardinality of the smallest dominating set of a graph, is noted $\gamma(G)$

Domination Examples

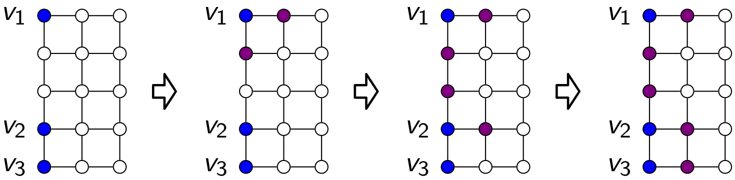

Figure: $S = \{v_1, v_2, v_3\}$ is not a dominating set

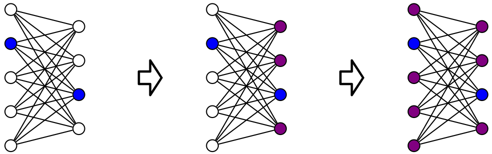

Figure: S is a dominating set

Zero Forcing

- ZERO_FORCE(G, S)

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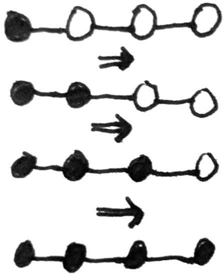
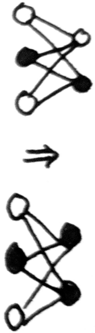
Assuming S ⊆ V
while S has not been changed do
    for v ∈ S do
        if Exactly one empty vertex in N(v) then
            Fill in that vertex and add it to S
        end if
    end for
end while
```

- If, after the zero forcing has been performed, $S = V$, then the initial set S is a *zero forcing set* of G.
- The *zero forcing number* of a graph is noted F(G)

Zero Forcing Examples



S is a zero forcing set.



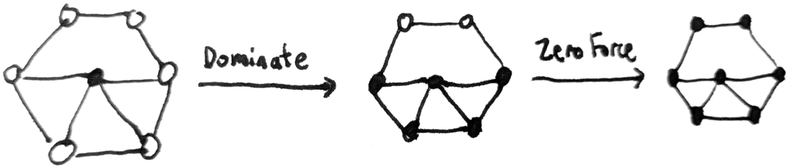
S is not a zero forcing set.

Power Domination

- POWER_DOMINATE(G, S)
  1: DOMINATE(G, S) {S will be updated}
  2: ZERO_FORCE(G, S)

- If, after the power domination has been performed, $S = V$, then the initial set S is a *power dominating set* of G.
- The *power domination number* of a graph is noted γ_p

Power Domination Example



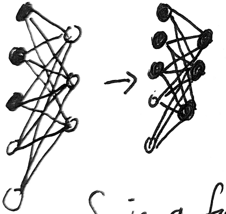
S is a power dominating set.

Failed Power Domination

- $\bar{\gamma}_p$
- The *failed power domination number* of a graph is the cardinality of the **largest** set that does not power dominate the graph.

Failed Power Domination Example

$$\bar{\gamma}_p(K_{5,3}) = 3$$



S is a failed power dominating set.

If you add just one more vertex to S, it will be a power dominating set.

PMUs

- Phasor Measurement Units
  - Monitor electric power networks
  - Expensive

- $\bar{\gamma}_p(G) + 1$ is the number of PMUs that you can put **anywhere** on the graph and monitor the entire graph.

Outline

Program and Results

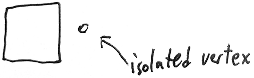
N=$\|G.V\|$	Number of graphs with $\bar{\gamma}_p$(G)=0	Total possible graphs with N vertices $=2^{(N\ choose\ 2)}$
0	1	1
1	1	1
2	1	2
3	4	8
4	22	64
5	218	1024
6	3868	32768
7	108136	2097152

Extreme Values

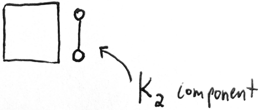
Let n=|G.V|

Let □ denote some graph.

$\bar{\gamma}_p(G) = n-1$ if and only if G has an isolated vertex.

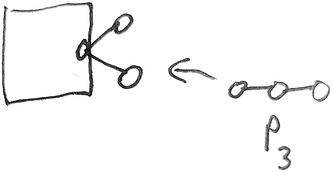


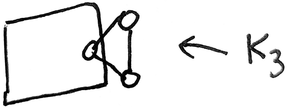
$\bar{\gamma}_p(G) = n-2$ if and only if G contains K_2 as a component and no isolated vertices



Extreme Values

- $\bar{\gamma}_p(G) = n - 3$ if and only if G contains no components that are isolated vertices or K_2 and G contains a copy of
  - P_3 where only the middle vertex may be adjacent to other vertices in the $V(G)$, or
  - K_3 where at most one of the vertices may be adjacent to other vertices in $V(G)$





Complexity

FAILED POWER DOMINATING SET (FPDS)
Instance: a graph $G = (V, E)$ and a positive integer k
Question: Does G have a proper stalled subset of cardinality at least k?

Theorem

FPDS is NP-complete.

The proof consists of

1. Show that FPDS is NP-hard by constructing a polynomial reduction from the independent set problem (does G contain an independent set of cardinality ℓ)?
2. It is known that determining whether or not set $S \subseteq V$ with $|S| \leq k$ is a power dominating set is verifiable in polynomial time.

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