FAILED POWER DOMINATION ON KNÖDEL GRAPHS

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Abstract

Let G be a simple graph with vertex set V and edge set E, and let $S \subseteq V$. The *open neighborhood* of $v \in V$, $N(v)$, is the set of vertices adjacent to v; the *closed neighborhood* is given by $N[v] = N(v) \cup \{v\}$. The *open neighborhood* of S, $N(S)$, is the union of the open neighborhoods of vertices in S, and the *closed neighborhood* of S is $N[S] = S \cup N(S)$. The sets $\mathcal{P}^i(S), i \geq 0$, of vertices *monitored* by S at the i^{th} step are given by $\mathcal{P}^0(S) = N[S]$ and $\mathcal{P}^{i+1}(S) = \mathcal{P}^i(S) \bigcup \{w : \{w\} = N[v] \backslash \mathcal{P}^i(S)$ for some $v \in \mathcal{P}^i(S)\}$. If there exists j such that $\mathcal{P}^j(S) = V$, then S is called a *power dominating set*, PDS, of G.

We introduce and discuss the *failed power domination number* of a graph G, $\bar{\gamma}_p(G)$, which is the cardinality of the largest subset of V that is not a PDS. We show cases where $\bar{\gamma}_p(G)$= $|V|$ through $|V| - 3$. Also shown are results for some named graphs, such as Knödel graphs.

Definitions and Notation

G = (V,E):= A graph, G, with set of vertices V, and set of edges E. Vertices are connected via edges. We only work with undirected graphs, which means that all of the edges are bidirectional.

u, v: arbitrary vertices u and v

N(v):= Set of neighbors of a vertex.

Domination

DOMINATE(G, S):

```
1: Assuming S ⊆ V                // Start with some initial set
2: for v ∈ S do                  // For every vertex in that set...
3:   for u ∈ N(v) do             // Check the vertex's neighbors
4:     if u ∉ S then             // If the neighbor is not already
5:       S = S ∪ {u}             // in that set then add it to the set
6:     end if
7:   end for
8: end for
```

- After domination has been performed, if $S = V$, then the initial set S is a *dominating set* of G
- The *domination number* of a graph, or the cardinality of the smallest dominating set of a graph, is noted $\gamma(G)$

Domination Examples

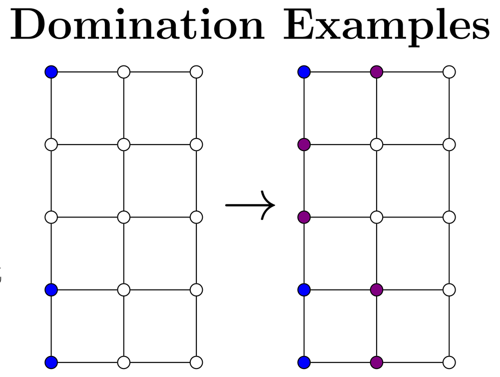


Figure 1: S is not a dominating set.

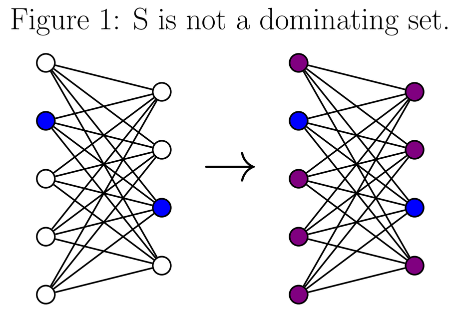


Figure 2: S is a dominating set.

Zero Forcing

ZERO_FORCE(G, S):

```
1: Assuming S ⊆ V
2: while S has not been changed do
3:   for v ∈ S do
4:     if Exactly one empty vertex in N(v) then
5:       Fill in that vertex and add it to S
6:     end if
7:   end for
8: end while
```

- If, after the zero forcing has been performed, $S = V$, then the initial set S is a *zero forcing set* of G.
- The *zero forcing number* of a graph is the cardinality of the smallest zero forcing set of a graph, and is noted F(G)

Zero Forcing Examples

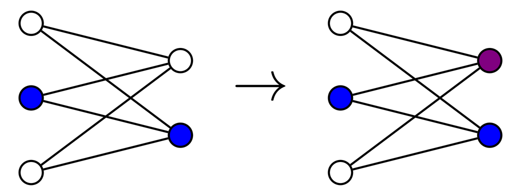


Figure 3: S is not a zero forcing set.

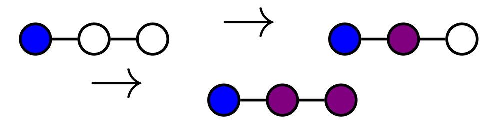


Figure 3: S is a zero forcing set.

Power Domination and Failed Power Domination

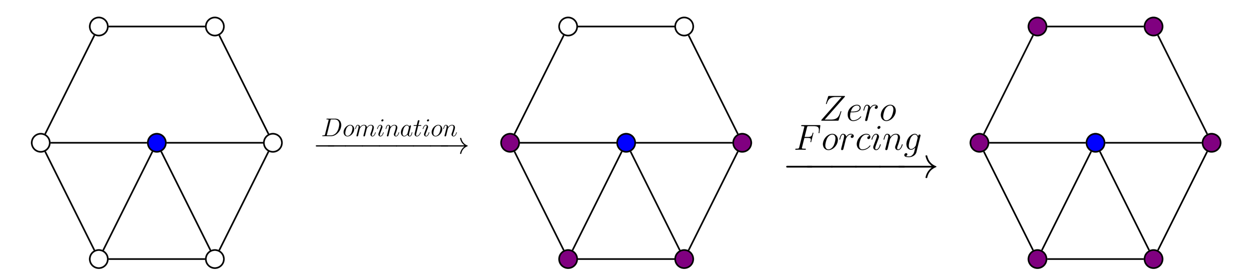
Power Domination

POWER_DOMINATE(G, S) {S will be updated at each step}

```
1: DOMINATE(G, S)
2: ZERO_FORCE(G, S)
```

- If, after the power domination has been performed, $S = V$, then the initial set S is a *power dominating set* of G.
- The *power domination number* of a graph is the **minimum** cardinality of such a power dominating set and is noted $\gamma_p(G)$

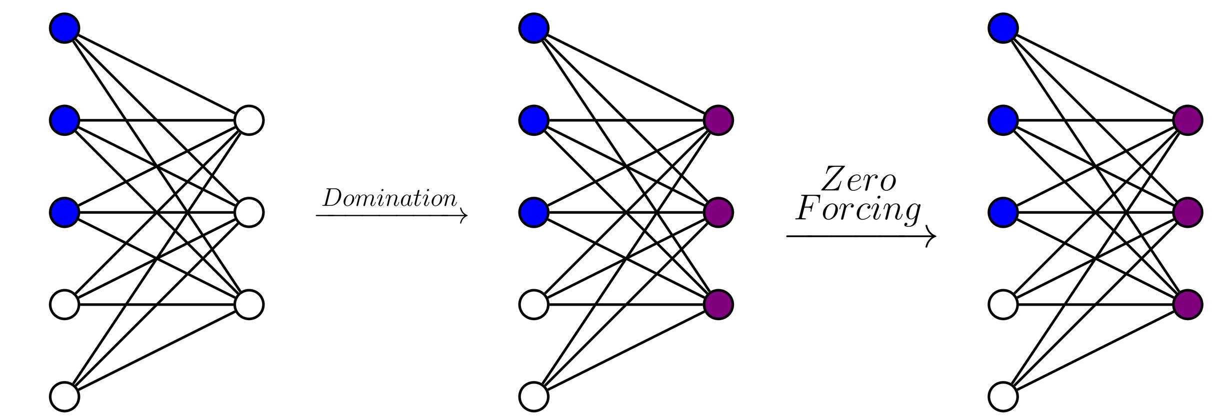
Power Domination Examples



Failed Power Domination

- $\bar{\gamma}_p$
- The *failed power domination number* of a graph is the cardinality of the **largest** set that does not power dominate the graph.

Failed Power Domination Examples



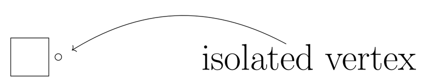
S is a failed power dominating set. If you add any more vertices to S, it will become a power dominating set.

Results

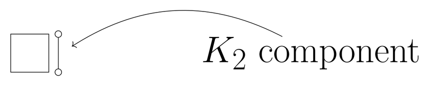
General Extreme Values

Let n=$|G.V|$ and □ denote a graph.

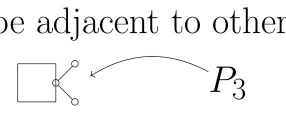
$\bar{\gamma}_p(G) = n - 1$ if and only if G has an isolated vertex.



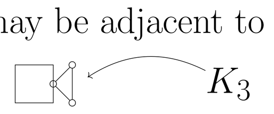
$\bar{\gamma}_p(G) = n - 2$ if and only if G contains K_2 as a component, and no isolated vertices.



- $\bar{\gamma}_p(G) = n - 3$ if and only if G contains no components that are isolated vertices or K_2 and G contains a copy of
  - P_3 where only the middle vertex may be adjacent to other vertices in the $V(G)$

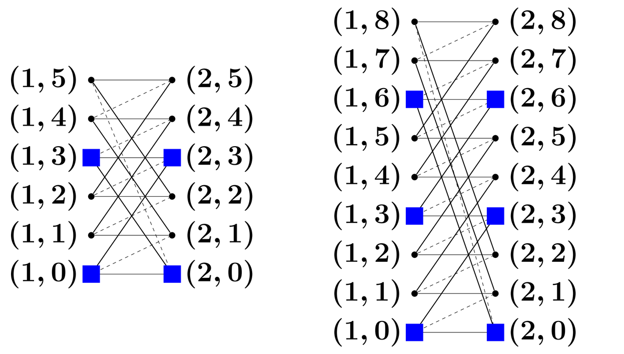


  - K_3 where at most one of the vertices may be adjacent to other vertices in $V(G)$



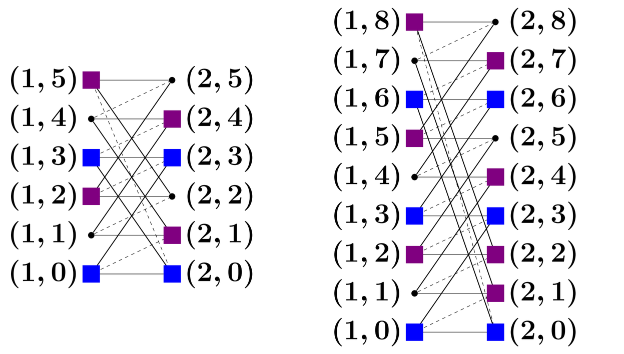
Knödel Graphs $W_{\#vertices,\ degree}$

In this case, we look at $W_{6k,3}$ where k $\geq$ 2. For example, $W_{6(2),3}$ and $W_{6(3),3}$ are shown on the right. The failed power dominating initial sets for consist of every third pair of vertices side by side, and are colored blue here.



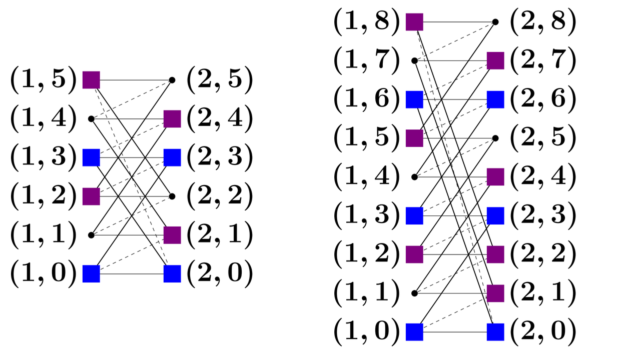
Knödel Graphs – Domination Step

Some vertices are dominated.



Knödel Graphs – Zero Forcing Step

We cannot fill in any more vertices. The Zero Forcing step is stalled and the initial set fails to power dominate.



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